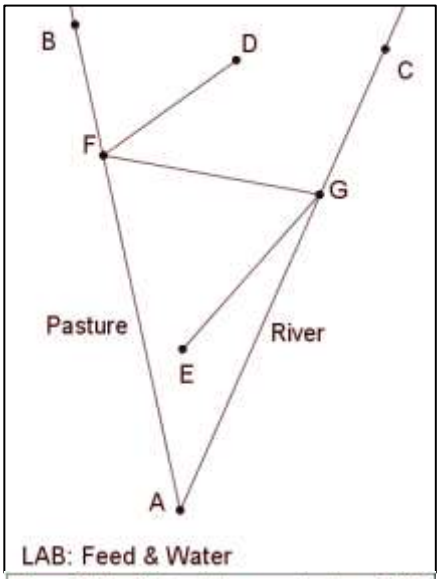
GGB Lab: The Feed & Water Problem

(revised for Geogebra from Exploring Geometry with The Geometer's Sketchpad Key Curriculum Press, 2012, pp 67)

OBJECTIVE: In this investigation you will determine the shortest multi-step path to help a rider, traveling from one location to another, feed and water his horse along the way following a shortest path.

SKECTH and INVESTIGATE: Open a new GGB file and save it as "LAB Feed & Water_1"
Answer the questions in your composition book as you complete the lab and questions are asked.

A rider is traveling from point D to point E between a river and a pasture. Before the rider gets to E, he wants to stop at the pasture to feed his horse and at the river to water his horse. What path should the rider take to minimize the distance the horse must walk?



LAB: Feed & Water

From D to F (start to pasture)	2.68
From F to G (pasture to river)	3.61
From G to E (river to finish)	3.38
Total MINIMUM distance:	9.67

1. Construct two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Show the label for each ray. Show the label for these rays. Choose object properties for these rays and rename the left ray "Pasture" and the right ray "River." See diagram.
2. Construct point D and E between the rays **as shown in the diagram**.
3. Construct segment DF from point D to the pasture **as shown in the diagram**.
4. Construct segment FG from the pasture to the river **as shown**.
5. Construct segment GE from the river to point E **as shown**.
6. Measure DF, FG and GE by using the distance tool and selecting each segment. (NOT each pair of segment endpoints).
7. Hide these distance measures to unclutter the sketch.
8. In the INPUT bar calculate TotalDistance = a + b + c. The algebra window will show Segment lengths called a, b, and c for these three distances. If your Algebra window does not show lengths a, b and c, then use the lengths that are listed in your Algebra window.
9. Create a textbox as shown here…… Remember to use the objects a, b, c, and TotalDistance so that the values in the textbox dynamically change.
10. Drag point F and G and try to minimize the total distance. You may have to drag each point several times.
11. Measure angles DFB, AFG, CGF and EGA. Selecting points in the clockwise order listed will shade angles that are less than 180 degrees.
12. Try again. Use the angle measures, as you drag points F and G, to make the path even shorter and minimize the total distance. You may have to drag each point several times.

QUESTION 1: What do you notice about the incoming and outgoing angles the path makes with the pasture and the river when the path is minimized?

EXPLORE MORE:

Open a new GGB file and save it as "LAB Feed & Water_2"

In this new lab you will construct your solution so that it shows the minimum distance path wherever points D and E are located. Follow these instructions:

13. In this new sketch, construct the rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and points D and E between them, just as you did earlier.
14. Reflect point D across the pasture to show image D'. Reflect point E across the river to show image E'.

QUESTION 2: What does the distance D'E' represent in the context of this problem scenario? Why?

15 Construct segment $\overline{D'E'}$

16 Construct the intersection points of segment $\overline{D'E'}$ with the Pasture ray and with the River ray. Point F and G will be the intersection points of segment $\overline{D'E'}$ with the rays.

17 Hide D', E', and $\overline{D'E'}$.

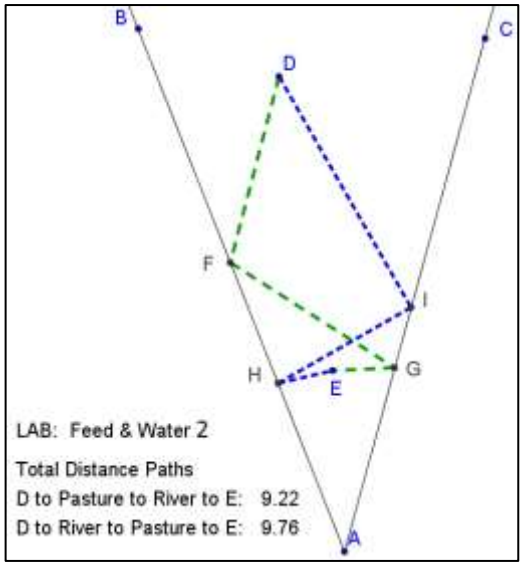
18 Construct segment $\overline{DF}$, $\overline{FG}$, and $\overline{GE}$. Change their object properties to green dashed and thicker segments.

19 Measure each segment $\overline{DF}$, $\overline{FG}$, and $\overline{GE}$. These distances will show up in the algebra window under Segments as a, b, and c.

20 Use the INPUT bar to find "PastureRiverSum=a+b+c" representing the sum of these three segments.

21 Drag point D and/or point E and the path will change automatically to minimize the distance.

22 Using a process similar to that described in steps 13-19, construct a minimal path that goes first from D to the river, then to the pasture, then to point E.



Repeating these steps:

13. Reflect point D across the river to show image D_1'. Reflect point E across the pasture to show image E_1'.
14. Construct $\overline{D_1'E_1'}$.
15. Hide D_1', E_1', and $\overline{D_1'E_1'}$.
16. Construct the intersection points of $\overline{D_1'E_1'}$ with the pasture ray and with the river ray. Point H and I will be the intersection points of $\overline{D_1'E_1'}$ with the rays.
17. Construct segment $\overline{DI}$, $\overline{HI}$, and $\overline{HE}$. (Note: H and I may be in opposite locations depending on the order in which you constructed these intersection points.)
18. Measure each segment $\overline{DI}$, $\overline{HI}$, and $\overline{HE}$. These distances will show up in the algebra window under Segments as d, e, and f.
19. Use the INPUT bar to find "RiverPastureSum=d+e+f" representing the sum of these segments.
20. Drag point D and/or E and the path will change automatically to minimize the distance.

QUESTION 3: Does it make any difference in the minimum sum value whether the rider feeds and then waters or waters and then feed his horse? What do you think the answer depends on?

23 It appears that one path is shorter than the other. Drag point D and E until the two different paths are the same length. You may have to drag each point several times.

24 Construct the angle bisector of angle BAC. Find the angle bisector tool in the same menu as perpendicular bisector.

QUESTION 4: What appears to be true about the location of D and E when the two paths are equal?

25 Hide the angle bisector.

26 Construct a line passing through D and E.

27 Drag point D and E so that they are no longer on the angle bisector. Notice the line intersects the river or the pasture. Examine the values for the total distance paths.

QUESTION 5: What is true about the line $\overleftrightarrow{DE}$ when the shortest path leads to the pasture first?

What is true about line $\overleftrightarrow{DE}$ when the shortest path leads to the river first?

LAB: The Feed and Water Problem Date_____

OBJECTIVE:

Find the minimum total distance on a multi-step route.

QUESTION 1: What do you notice about the incoming and outgoing angles the path makes with the pasture and the river when the path is minimized?

QUESTION 2: What does the distance D'E' represent in the context of this problem scenario? Why?

QUESTION 3: Does it make any difference in the minimum sum value whether the rider feeds and then waters or waters and then feed his horse? What do you think the answer depends on?

QUESTION 4: What appears to be true about the location of D and E when the two paths are equal?

QUESTION 5: What is true about the line $\overleftrightarrow{DE}$ when the shortest path leads to the pasture first?

What is true about line $\overleftrightarrow{DE}$ when the shortest path leads to the river first?