GGB Lab5b: Two Rivers: The Sum of the Distance from a Point to a Line

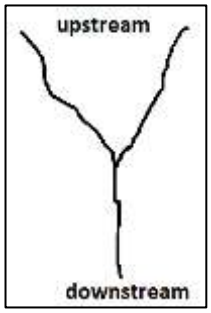
(revised for Geogebra from Exploring Geometry with The Geometer's Sketchpad Key Curriculum Press, 2012, pp 37)

Now that we know "The shortest distance from a point to a line is measured along the perpendicular line from the point to the line." We will use this idea to minimize the sum of two distances.

OBJECTIVE: In this investigation you will discover the minimum sum of the two distances from a point to two lines.

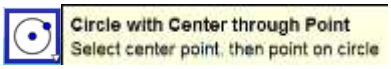
SKECTH and INVESTIGATE: Open a new GGB file and save it as "Lab5b Two Rivers Sum of Distances Pt to Line" Answer the questions in your composition book as you complete the lab and questions are asked.

We will model the following scenario in a Geogebra sketch. There is a sewage treatment plant at the point where two rivers meet. You want to build a house near the two rivers (upstream from the sewage plant, naturally), but you want the house to be at least 5 miles from the sewage plant. You visit each of the rivers to go fishing about the same number of times but being lazy, you want to minimize the amount of walking you do. You want the sum of the distances from your house to the two rivers to be minimal, that is, the smallest distance.



The big question is this. ***Where should you build your house?*** We will use the sketch to model the scenario and to answer the question.

1 Create a text box with the text: "**LAB 5b: Two Rivers**"

2 Construct a circle with center A and point B on the circle. Use 

and place point A in the graphics window, then place point B to define the circle.

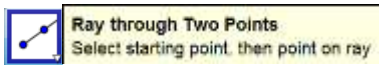
3 Rename point A, SewagePlant. Right click on point A, choose rename and type SewagePlant as one word and then change its color to red.

4 Rename point B, radius and change its color to red.

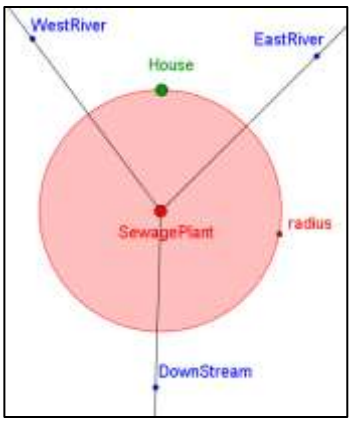
5 Right click on the circle. Change the object properties of the circle: the color to red and the opacity to 25.

6 Create a new textbox containing the text: "The circle represents the distance of 5 mile radius from the sewage plant."

7 Create a check box: "Show 5 mile radius" and select the circle, the radius point and the 5 mile radius textbox.

8 Construct three rays from SewagePlant to point A, B and C. Use 

to construct a ray from the sewage plant to point A representing the west river. Repeat to construct a ray from the sewage plant to point B representing the east river. Repeat to construct a ray from the sewage plant to point C representing the river downstream of the plant. Rename point A "WestRiver", B "EastRiver" and C "DownStream" making the sketch look like the diagram at the right.



9 Place a new point on the circle in between the West and East rivers. Rename this point "House" and change its size to 7 and its color to green.

10 Construct the specific line that shows the shortest distance from the house to the West river. (Remember Lab 5a? What line are you constructing to create the shortest distance?)

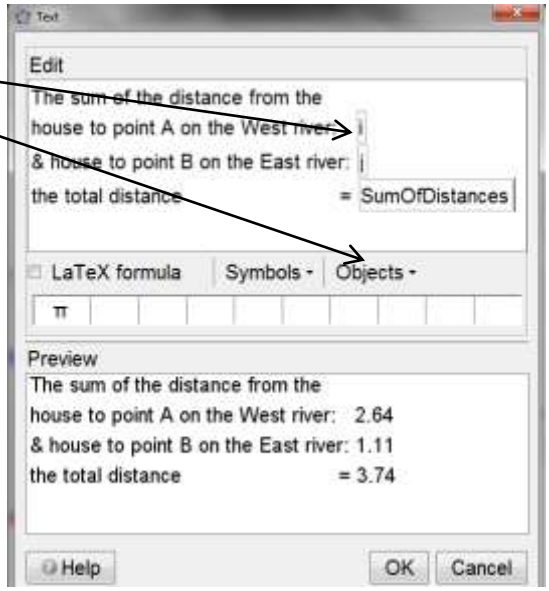
11 Construct the intersection point of this line and the west river ray.

12 Repeat step 10 and 11 to create the shortest distance from the house to the East river and the construct the intersection point.

13 Create a check box “Show Perpendiculars” and select each of the perpendicular lines from the house to point A on the West river and to point B on the East river. ESC.
14 Click on the check box to hide the perpendiculars.
15 Click on the check box to hide the circle.
16 Construct two segments from house to A and from house to B.
17 Measure these segment distances by choosing the distance tool and selecting each segment. A numerical value will appear next to each segment. If you have any letter label, undo, and re-measure by selecting the segment and not the two points.
18 Drag the green point representing the location of the House and observe the measurements. Remember our goal is to place the house so that the sum of the distances is the smallest.
19 Create a new check box to “Show Distances to Rivers.” Select these four objects: segment from House to point A, segment from House to point B, point A and point B.

It would be nice to see the sum of these two measures rather than to always have to add them up in your head. So let’s take care of that.

20 Look in the Algebra window for the list of Segment objects. You may see two segments: i=Segment [House, A] and j=Segment[House, B]. In the INPUT bar at the bottom of the page type the following exactly as you see it: SumOfDistances=i+j and press enter. If you look in the Algebra window under the Number objects you will see the variable SumOfDistances and a value that represents that sum.
21 Create a new textbox containing the following:
The text in the Edit window that appears in boxes is the variable object and you can get these from the drop down Objects menu.
Align the Objects vertically as shown here then press OK.
22 Show the circle and show the segment distances. Drag the green House point and find location that is the smallest sum of the distances to the rivers.



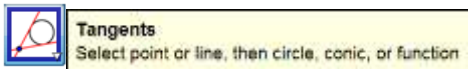
QUESTIONS for your consideration: Record your answers in your composition book.

1. Is there more than one location that will yield a minimum distance? Describe where your house can be located to minimize your walking distance to the rivers.
2. What do you think about building your house in this location?
3. What path do you need to take to get to the West and East rivers if your house is located here?
4. What are the advantages to this location?
5. What are the disadvantages to this location?

EXPLORE MORE:

You have decided to revise the requirement for locating your house. Although you love fishing on the river, you would prefer to fish farther upstream away from the sewage plant and outside the five mile radius. You are looking for a new location exactly 5 miles from the sewage plant. You would like to find a location for your house that allows you to walk to the East and West rivers and minimize the total walking distance but allow you to fish outside the five mile radius. Complete the following steps to help decide where should you build your house?

23 Hide the Distances to the Rivers objects using the check box you created.

24 Construct a Tangent line to the circle through the point House. Use 

select House and then select the circle. Drag point House around the circle and observe the behavior of the tangent line. Drag House to a location between the two rivers.

25 The tangent line intersects the two rivers. Construct points C and D at the intersection of the tangent line and the two rivers.

26 Hide the tangent line.

27 Construct segment CD, change its object properties to a dashed and green.

28 Measure the length of the line segment CD. This line segment represents the sum of the distance from your house to the West and East rivers.

29 Drag the House point. Compare the distance from the tangent segment to the total distance found using perpendicular segments in the previous part of the lab.

QUESTION 6: Will the tangent segment distance ever be as small as the perpendicular segments? WHY?

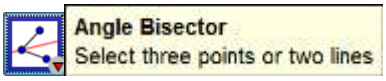
30 Construct radius segment from the point SewagePlant to the point House.

31 Measure the two central angles using the points House-SewagePlant-C and using the points D-SewagePlant-House.

32 Drag the House point and find the location where the green-dashed tangent segment distance is the smallest.

QUESTION 7: What do you observe about the two central angles when the tangent segment distance is the smallest?

QUESTION 8: Where will you locate your House so that the paths you must walk to the rivers are outside the 5 mile radius and the sum of the distances to each river is the smallest?

33 Construct the Angle Bisector of angle C-SewagePlant-D. Use 

and select point C then SewagePlant then point D.

QUESTION 9: Drag point House again. Where is House located when you find a minimum tangent segment distance?