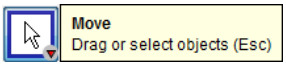
GGB Lab: Euclid's Proposition 1 – An Equilateral Triangle

(revised for Geogebra from Exploring Geometry with The Geometer's Sketchpad Key Curriculum Press, 2012, pp 11)

Euclid, a Greek mathematician born around 300 B.C.E., wrote a book called the *Elements*, upon which most school geometry books are still based. All of the geometry in the *Elements* is built up sequentially from a few simple constructions and postulates. Each new property that Euclid presents, or new figure that he constructs, is based on properties he has demonstrated previously. The construction that starts it all is the equilateral triangle. Countless other constructions in the *Elements* depend on being able to construct and equilateral triangle with a compass and a straightedge.

OBJECTIVE: In this activity you will construct an equilateral triangle using Geogebra's freehand tools – the equivalents to Euclid's compass and straightedge.

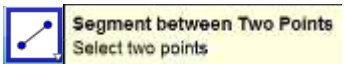
SKECTH and INVESTIGATE: Create a "MATH or GEOMETRY" folder on your computer. Inside the folder create a new folder and name it "GEOGEBRA". Open a new GGB file and save it as "Lab1 Euclids Equilateral Triangle" Answer the questions in your composition book as you complete the lab and questions are asked.

1 ***NOTE:*** Get in the habit of pressing ESC (the escape button on your keyboard) to get back to the move tool 

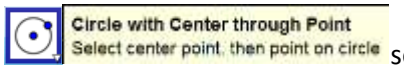
especially after using other drop-down menu commands.

2 ***NOTE:*** When you make a mistake, it is easy to undo. Press Ctrl-Z or Command-Z either once or repeatedly to return to a previous state of your sketch.

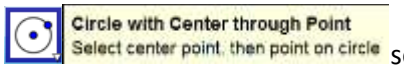
Now you will begin the work to make Euclid's dynamic sketch. Follow these instructions which will always first **state what you need to do** and then second **state how you are to do it**. So if you read you will be successful.

3 Construct segment AB. Use 

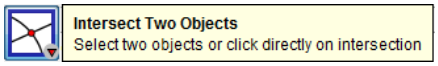
left click in graphics window to place point A, move and left click again to place point B.

4 Construct circle AB, using point A as the center and point B as the endpoint of the radius. Use 

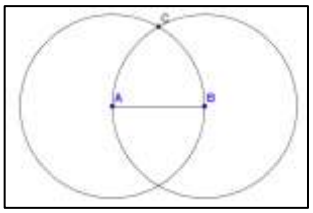
select point A then point B.

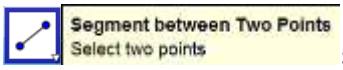
5 Construct circle BA, using point B as the center and point A as the endpoint of the radius. Use 

select point B then point A.

6 Construct point C, the intersection of the two circles as shown in the diagram. Use 

. When you move the cursor over the intersection both circles should be bolder, click to place point C. Drag point A or point B to be sure that C remains at the intersection of the circles.



7 Construct segment AC and segment BC. Use 

select point A then C, and repeat, select point B then C.

QUESTION 1: Drag point A or point B. What happens to your triangle? Does it appear to remain equilateral?

QUESTION 2: Explain why the triangle is always equilateral. (HINTS: What roles do circles play in the construction? How are they related to one another? How are the sides of the triangle related to the circles?)

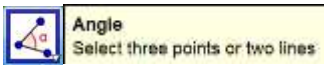
8 Change the line style of the circles to a faint dotted curve. Right click on each circle, choose Object Properties, in the Style menu select 

. Also change the color of the dashed circles to a color of your choice.

9 Create a Check Box that will allow you to show or hide the circles. Use 

and click in the white space of the graphics window. When the dialogue box opens, in the Caption box type "Show-Hide Circles" then left click on the circle with center at A, then left click on the circle with center at B, finally select apply at the bottom of the dialogue box. Press ESC. Click on the check box to show-hide the circles. They should disappear and reappear. Hide the circles so that only the triangle is showing and then continue with the next steps.

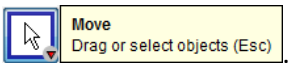
10 Change the color of your triangle segments to a color of your choice, but different from the color of your circles.

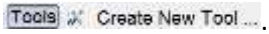
11 Measure the three angles of the triangle. Use 

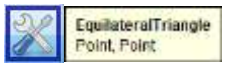
select point B, A, then C. Select C, B, then A. Select A, C, then B. If you select the three points in a clockwise order the angle measure will appear inside the angle of the triangle. (If you select points in a counter-clockwise order the angle measure will appear outside the angle of the triangle but may be changed to inside by editing the Object Properties of the angle measure. Ask if you need instruction.)

QUESTION 3: Drag point A or point B. Make a conjecture about the angles in an equilateral triangle. A conjecture is an educated statement based on your observations.

EXPLORE MORE:

12 Create a custom tool for the equilateral triangle. Press ESC to get back to the move tool 

. In the graphics window, highlight and select your equilateral triangle. Then from the TOOLS menu select Create New Tool… 

. When the dialogue box opens you will see three tabs. The first Output Objects, select next. The second Input Objects, select next. The third Name & Icon. To name the Custom Tool type "EquilateralTriangle" in place of tool. Select FINISH. A new icon 

shows up at the right side of the tool icons.

13 Use the new Equilateral Triangle tool to create more equilateral triangles in this sketch. Select two points on your triangle ABC in a **clockwise** order to construct a second triangle that shares a side with the first. If the new triangle overlaps the previous triangle it means you selected points in a counter-clockwise order. Press CTRL Z and select the points in a **clockwise** order. Keep using the tool on different pairs of vertices and fill the screen with a tessellation of equilateral triangles.

QUESTION 4: Why do the triangles fill the plane without any gaps or overlaps? (Think about how many triangles immediately surround one vertex and what are the angle measures that surround that one vertex.)