

GGB Lab: Dilations by Scale Factor about Center

(revised for Geogebra from Exploring Geometry with The Geometer's Sketchpad Key Curriculum Press, 2012, pp 286-287)

OBJECTIVE: In this investigation you will learn about the characteristics of dilations that define similar polygons.

You have learned about transformations that are **isometries**, that is a transformation such as a translation, reflection or rotation that maintains size and shape. When figures are **similar** they have the same shape but do not necessarily have the same size. A **dilation** is a transformation that preserves shape. In this investigation you will use dilation to discover principles of similarity and write a definition of similar polygons.

INVESTIGATE USING the PRE-CONSTRUCTED LAB:

Open the GGB file: “Dilations by Scale Factor about Center” and save it in your GGB folder. Answer the questions in your composition book as you complete the exploration when questions are asked.

- 1 In the graphics window, locate point A. Point A is the **center of the dilation** for the quadrilateral pre-image BCDE.
- 2 Check the checkbox “DilationByIntegerScaleFactor.” Notice the **pink** quadrilateral image B’C’D’E’ that has the same shape as pre-image BCDE but is a different size. B’C’D’E’ is a dilation of the pre-image BCDE.
- 3 Drag the slider labeled “Integer Scale Factor: n” and observe the location and size of the dilated quadrilateral image B’C’D’E’.

QUESTION 1: Using “Integer Scale Factor: n” Slider

- A What slider value will result in the quadrilateral image B’C’D’E’ being the same size as the pre-image BCDE and on the **same side** of the center of dilation as the pre-image?
 - B What slider value will result in the quadrilateral image B’C’D’E’ being the same size as the pre-image BCDE and on the **opposite side** of the center of dilation as the pre-image?
 - C What range of slider values will result in the quadrilateral image B’C’D’E’ being larger than the pre-image and on the **same side** of the center of dilation as the pre-image?
 - D What range of slider values will result in the quadrilateral image B’C’D’E’ being larger than the pre-image and on the **opposite side** of the center of dilation as the pre-image?
- 4 Check the checkbox “ShowDilationRaysfromA.” Notice the dashed lines pass from the center point A through each of the pre-image vertices and go on to pass through each of the image vertices.

QUESTION 2:

- A At what single point do the dilation rays or lines intersect?
 - B How could you find the center of dilation when given a pre-image and a dilated image?
- 5 Check the checkbox “Show Ratios for Scale Factor n.” At the bottom of the graphics window there appears a textbox with pink text which shows two sets of ratios that are equal to the scale factor. Examine the ratios that are shown. These ratios indicate how to find the scale factor when you have two similar figures and the center of dilation.

QUESTION 3: Remember a ratio is a fraction. There are two sets of ratios that determine the dilation scale factor.

Examine the ratios shown in the textbox then fill in the blanks for each ratio below.

- 1) Scale Factor = $\left(\frac{\text{Distance from the center of dilation to the image vertex}}{\text{Distance from center of dilation to } \underline{\hspace{2cm}}} \right)$
- 2) Scale Factor = $\left(\frac{\text{Distance of one side of image from vertex to vertex}}{\text{Distance of } \underline{\hspace{2cm}}} \right)$

- 6 Set the Integer Scale Factor n slider to -1 . Use the last icon and its drop down menu to Zoom In 4 times on point A.
- 7 Check the checkbox "DilationsByScaleFactorBetweenNeg1andPos1". Notice the **green** quadrilateral image $B''C''D''E''$ that has the same shape as pre-image BCDE but is a different size. $B''C''D''E''$ is also a dilation of the pre-image BCDE.
- 8 Drag the slider labeled "Real Scale Factor: a " and observe the location and size of the dilated quadrilateral image $B''C''D''E''$.



QUESTION 4: Using "Real Scale Factor: a " Slider

- A What slider value will result in the quadrilateral image $B''C''D''E''$ being the same size as the pre-image BCDE and on the **same side** of the center of dilation as the pre-image?
 - B What slider value will result in the quadrilateral image $B''C''D''E''$ being the same size as the pre-image BCDE and on the **opposite side** of the center of dilation as the pre-image?
 - C What range of slider values will result in the quadrilateral image $B''C''D''E''$ being smaller than the pre-image and on the **same side** of the center of dilation as the pre-image, (between the center of dilation and the pre-image)?
 - D What range of slider values will result in the quadrilateral image $B''C''D''E''$ being smaller than the pre-image and on the **opposite side** of the center of dilation as the pre-image?
- 9 Check the checkbox "Show Ratios for Scale Factor a ." At the bottom of the graphics window there appears a textbox with green text which shows two sets of ratios that are equal to the scale factor. Examine the ratios that are shown. These ratios indicate how to find the scale factor when you have two similar figures and the center of dilation.

QUESTION 5: . There are two sets of ratios that determine the dilation scale factor. Examine the ratios shown in the textbox then fill in the blanks for each ratio below.

- 1) Scale Factor = $\left(\frac{\text{Distance from the center of dilation to the image vertex}}{\text{Distance from center of dilation to } \underline{\hspace{2cm}}} \right)$
- 2) Scale Factor = $\left(\frac{\text{Distance of one side of image from vertex to vertex}}{\text{Distance of } \underline{\hspace{2cm}}} \right)$
- 3) Are these ratios different from question 3 that you answered previously?

QUESTION 6: A scale factor of -1 results in a dilation image that is equivalent to a single isometric transformation. What single isometric transformation is equivalent to a dilation by -1 ?

QUESTION 7: What appears to be true **about the slopes** of corresponding sides of the similar pre-image and and image dilations? If this is true about the slopes then what can you conclude about the corresponding sides?

QUESTION 8: What appears to be true about corresponding angles of the pre-image and the image dilations? It is because of this relationship that the pre-image and image are called **similar** and they have the same shape.