Geometry December 2, 2013 Name___________________________

Fibonacci Sequence and the Golden Ratio PHI period 1 2 3 5 6

The Fibonacci Sequence is a list of numbers that begins with 1, 1, ...
Every number that follows is the sum of the previous two numbers in the sequence.

1. Complete the first 24 terms of the Fibonacci sequence in the table below

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
a_n	1	1																						

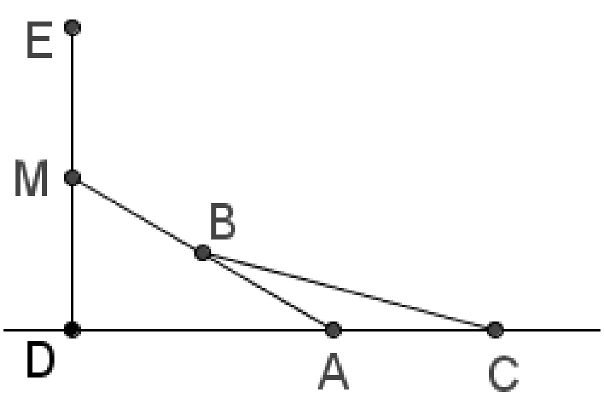
2. Use a calculator and find the ratio of consecutive terms of the Fibonacci sequence. Write your answers as a decimal using five decimal places.

Ratio	$\frac{a_2}{a_1}$	$\frac{a_3}{a_2}$	$\frac{a_4}{a_3}$	$\frac{a_5}{a_4}$	$\frac{a_6}{a_5}$	$\frac{a_7}{a_6}$	$\frac{a_8}{a_7}$	$\frac{a_9}{a_8}$	$\frac{a_{10}}{a_9}$	$\frac{a_{11}}{a_{10}}$	$\frac{a_{12}}{a_{11}}$	$\frac{a_{13}}{a_{12}}$	$\frac{a_{14}}{a_{13}}$	$\frac{a_{15}}{a_{14}}$
$\frac{a_{n+1}}{a_n}$ … to 5 Decimal Places														

3. What do you notice about the ratio of consecutive terms as the Fibonacci numbers get larger and larger?

4. Find the ratio of $\frac{a_{21}}{a_{20}} =$ Find the ratio of $\frac{a_{24}}{a_{23}} =$

The value that these ratios appear to be converging on is a number called PHI ϕ or Φ = 1.618 0339 887 and was described by Johannes Kepler as one of the "two great treasures of geometry." The other is the Pythagorean Theorem.



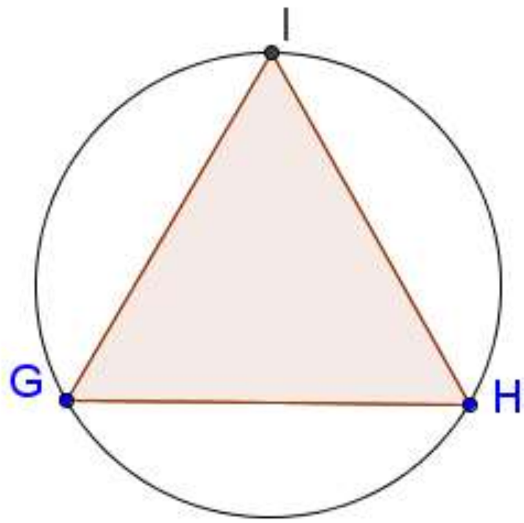
5. In the diagram DE and DC are perpendicular.
Length DE = MA = BC
M is midpoint of DE and B is midpoint of MA.

Using a centimeter ruler, measure and record each length.

Length DA = __________ Length DC =__________

Find the ratio of DC : DA and record the answer to 5 decimal places: ____________________

http://www.goldennumber.net/geometry/
http://www.goldennumber.net/spirals/ http://www.goldennumber.net/nautilus-spiral-golden-ratio/

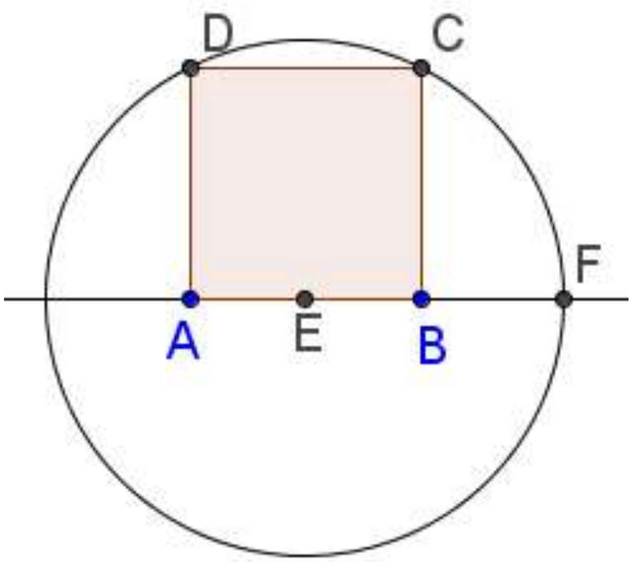


6. In the diagram, GHI is an equilateral triangle.

Use a compass to locate point M the midpoint of GI.
Use a compass to locate point N the midpoint of HI.
Draw a segment from M through N until it intersects the circle at point P.
Measure in centimeters.

Length MN = __________ Length MP =__________

Find the ratio of MP : MN and record the answer to 5 decimal places: ____________________________

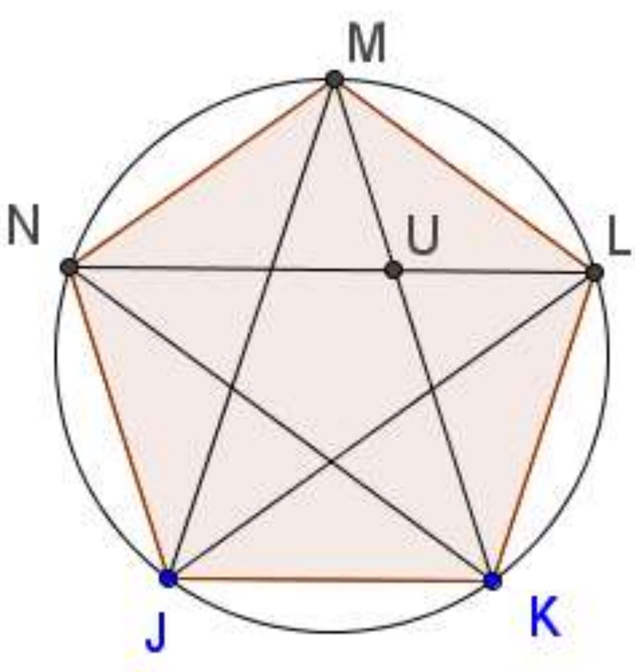


7. In the diagram, ABCD is a square constructed in a semicircle with center at E.

Measure in centimeters.

Length AB = __________ Length AF =__________

Find the ratio of AF : AB and record the answer to 5 decimal places: _____________________



8. In the diagram, JKLMN is an equilateral pentagon inscribed in a circle. Measure in centimeters.

Length NU = __________ Length NL =__________

Find the ratio of NL : NU and record the answer to 5 decimal places: _____________________

9. The Fibonacci sequence has many interesting properties. Check this one out. If you add up the squares of consecutive Fibonacci numbers, starting at the beginning, the sum will always be the product of the last number and the sum of the last two numbers. Here is an example:

$$1^2+1^2+2^2+3^2=3(2+3)$$

List two more equations demonstrating this property:

1)____

2)____

http://www.goldennumber.net/geometry/
http://www.goldennumber.net/spirals/ http://www.goldennumber.net/nautilus-spiral-golden-ratio/